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An analytical model for the heat and mass transfer processes in indirect evaporative cooling with parallel/counter flow configurations

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Abstract

This paper aims at developing an analytical model for the coupled heat and mass transfer processes in indirect evaporative cooling under real operating conditions with parallel/counterflow configurations. Conventionally, one-dimensional differential equations were used to describe the heat and mass transfer processes. In modeling, values of Lewis factor and surface wettability were not necessarily specified as unity. Effects of spray water evaporation, spray water temperature variation and spray water enthalpy change along the heat exchanger surface were also considered in model equations. Within relatively narrow range of operating conditions, humidity ratio of air in equilibrium with water surface was assumed to be a linear function of the surface temperature. The differential equations were rearranged and an analytical solution was developed for newly defined parameters. Also, performances with four different flow configurations were briefly discussed using the analytical model. Through comparison, results of analytical solutions were found to be in good agreement with those of numerical integrations.

Keywords: Indirect evaporative cooling; Parallel/counterflow configurations; Coupled heat and mass transfer; Analytical model

1. Introduction

Indirect evaporative coolers can be utilized to cool air or other fluid with wet surface heat exchangers. The surface of the cooling air passages is wetted by spray water (also named recirculation water), so that water film evaporates into the cooling air and decreases the temperature of the wetted surface. The primary air or other process fluid flows in the alternative passages and is cooled by indirect contact with the spray water film through the separating wall of the heat exchangers.

Pescod [1] proposed a plastic plate heat exchanger (PPHE) design for indirect evaporative cooling of air. Maclaine-cross and Banks [2] developed by analogy from published solutions for dry bulb temperature in dry surface heat exchangers an analytical solution to the indirect evaporative cooling processes based on the following assumptions: moisture contents of air in equilibrium with the water surface were assumed as a linear function of the water surface temperature; the evaporating water film was stationary and continuously replenished at its surface with water at the same temperature; Lewis relation is satisfied. Predictions of the efficiencies of Pescod's plate heat exchangers with wet passages were found to be significantly greater than reported measurements. Thus, poor wetting of Pescod's plastic plates was suspected. Kettleborough and Hsieh [3] used a simple wettability factor to describe the effect of incomplete wetting. The changes of the spray water temperature along the heat exchanger surface were also taken into consideration. Numerical analysis was utilized to study the thermal performances of a counterflow indirect evaporative cooler unit. The agreement between the calculated and measured performance data was improved qualitatively. Erens and Dreyer [4] reviewed three different models: (1) Poppe model—considering the over saturation

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Nomenclature

a	thermal diffusivity coefficient [m^2/s]
A	total interface area for heat transfer in the heat exchanger unit [m^2]
$\mathbf{A}$	coefficients matrices defined in Eqs. (16) and (21)
B_1–B_3	constants in Eqs. (7), (10) and (19), etc.
$\mathbf{B}$	coefficients matrix defined in Eq. (30)
c_p	specific heat capacity [kJ/kg °C]
c_{pa}	specific heat capacity of dry air [kJ/kg °C]
c_{pm}	specific heat capacity of moist air $c_{pm} = c_{pa} + w_a c_{pv}$ [kJ/kg °C]
C_w^*, C_f^*	water to air and fluid to air heat capacity rate ratios, respectively
C_1–C_3	coefficients in Eq. (24)
d, e	constants in Eq. (1)
D	mass diffusivity coefficient [m^2/s]
$\mathbf{e}(NTU_x)$	independent variable vector defined in Eq. (28)
$\mathbf{E}$	identity matrix
$\mathbf{F}$	coefficients matrix in Eq. (35)
$\mathbf{g}$	constants vector in Eq. (35)
h	convective heat transfer coefficient [kW/m^2 °C]
h_D	convective mass transfer coefficient [kg/m^2 s]
$\bar{h}_{fg}$	a normalized heat of evaporation at reference condition (0 °C)
h_{fg}^0	evaporation heat of water at reference temperature (0 °C) [kJ/kg]
i_a	specific enthalpy of moist air [kJ/kg]
i_v	specific enthalpy of water vapor at water film temperature [kJ/kg]
$\mathbf{K}$	coefficients matrix in Eq. (24)
L_a, L_f, L_w	flow lengths in model exchanger in directions of cooling air, fluid stream and spray water, respectively
Le	Lewis number a/D
Le_f	Lewis factor
m	mass flow rate (kg/s)
NTU	number of heat transfer units
NTU_x	dimensionless coordinate defined as $dNTU_x = NTU\, d\bar{x}_w$
$\mathbf{P}$	coefficients matrix in Eq. (36)
$\mathbf{q}$	constants vector in Eq. (36)
r	a ratio of sensible heat transfer capacitances
R_{cv}, R_{cw}	water vapor and liquid water to dry air specific heat capacity ratios, respectively
t	temperature [°C]
U_0	overall heat transfer coefficient between fluid and water film [kW/m^2 °C]
v	a symbol that may represent any variable discussed such as temperatures, humidities, etc. in the definition of relative errors for the analytical model
w	humidity ratio of moist air [kg/kg(a)]
x_a, x_f, x_w	space coordinates in model exchanger originated from the inlets of cooling air, fluid stream and spray water, respectively
$\mathbf{Y}$	function variable vector defined in Eqs. (15) and (20)

Greek symbols

Δ	change of or difference between parameters
δ_a, δ_f	flow direction indicators for cooling air and fluid stream in relative to water film flow, respectively
ε	effectiveness
ϑ	dimensionless temperatures defined in Eq. (12)
λ_1–λ_3	roots of the characteristic equation
σ	surface wettability

Subscripts

a	air or on air side
an	refer to the results of the analytical model
asw	moist air in equilibrium with water surface
B	bottom position
f	fluid or on fluid side
i	inlet
max	maximum value
min	minimum value
mean	mean value
nu	refer to the results of numerical integration of the differential model
o	outlet
T	top position
v	water vapor
w	water film
wb	wet bulb temperature of air
x	local position

in secondary air; (2) Merkel model—can be derived from Poppe model by assuming a Lewis factor of unity and negligible effect of spray water evaporation and also implying that the secondary air never becomes over-saturated; (3) Simplified model—assuming that the recirculation water (spray water) temperature is constant through out the cooler. The simplified model was recommended for the evaluation of smaller systems and for initial design purpose. However, the more sophisticated methods should be used for more accurate performance prediction.

In an indirect evaporative fluid cooler or evaporative condenser, the process fluid or refrigerant flows inside the heat exchanger tubes. Most of the models given in the literature prior to 1960 were derived by assuming a constant or a theoretically constant spray water temperature [5], similar to that reviewed by Erens and Dreyer [4] and were still utilized in some recent studies for interpretation of experimental data for getting the relevant heat and mass transfer coefficients [6,7]. Parker and Treybal [8] presented an analytical method for evaporative liquid coolers which takes into consideration the effect of the spray water temperature variation based on the Lewis relation and the assumptions of linear air saturation enthalpy and negligible effect of water evaporation. However, an experimental study of the heat and mass transfer coefficients by Hasan and Siren [7] showed that heat and mass transfer analogy gives lower values of mass transfer coefficients than those found from measurements. Peterson et al. [5] applied transfer coefficients calculated according to Parker and Treybal's correlation and solved the differential equations in the same manner as described by Parker and Treybal to estimate the performance of plain tube evaporative condensers. Mizushina et al. [9] presented two methods of heat calculation in coolers: one simplified, constant temperature of water spraying the tubes was assumed and another, which took into account the variation of that temperature in an exchanger. Another kind of approximation is to neglect the effect of enthalpy change of spray water [10,11]. This assumption is in effect identical to the stationary water film assumption made by Maclaine-cross and Banks [2]. Comparative study of different models showed that the prediction of the performance of an exchanger with this approximation gives the highest values [12]. More detailed models were presented by Zalewski [13] for evaporative condensers and by Zalewski and Gryglaszewski [14] for evaporative fluid coolers. In these models, the nonlinear relation of saturation air enthalpy with temperature and the effect of water evaporation were incorporated in the differential equations. Though heat and mass transfer analogy was still utilized to calculate the mass transfer coefficients between water surface and the cooling air from known heat transfer coefficients, the values of Lewis factor were not necessarily set to be equal to unity. In all of the above models, the mass flow rate of spray water are always assumed to be sufficient large so that the surface wetting is complete. However, the results of experimental study by Facao and Oliveira [12] showed that incomplete wetting might occur with relatively small mass flow rate of spray water.

On above discussion, we can see that indirect evaporative air or fluid coolers work in a similar way for heat and mass transfer processes and can be described with the same set of differential equations. The differences are in their heat and mass transfer coefficients and heat capacities of the fluid streams for different coolers. Merkel approach or other simplified methods will sacrifice accuracy for simplicity of solution. Detail models that apply to incomplete surface wetting condition with non unity values of Lewis factor and take into consideration the effects of spray water evaporation, spray water temperature variation and spray water enthalpy change along the heat exchanger surface will give more accurate results. However, analytical solution of such kind of detailed models was not found in literature. The objective of this paper was to develop an analytical solution for such kind of models for indirect evaporative cooler with parallel/counterflow configurations. Performances with four different flow configurations were briefly discussed.

2. Physical model

In indirect evaporative cooler, recirculation water is sprayed onto the top of the heat exchanger and flows along the wall surfaces of cooling air passages. The primary air or other process fluid flows in the alternative passages. For simplification of iteration, either the primary air or the process fluid will be named as fluid stream here after. Heat is transferred from the fluid stream to the cooling air through the heat exchanger wall and water film. The physical model can be shown schematically in Fig. 1(a). Due to high values of water surface tension, the wall surface of cooling air passages cannot be completely wetted with spray water and this leads to a reduced mass transfer area for film evaporation. In order to improve the model accuracy, a wettability factor will be utilized to describe the effect of incomplete wetting conditions. Also, Lewis factor is not necessary set as unity even for the uniformly wetted conditions [4].

However, as usually found in many previous researchers [2,3,13,14], the following assumptions are still adopted in this study

(1) zero wall, air thermal and moisture diffusivity in the flow directions;
(2) no heat transfer to the surroundings occurs;
(3) constant specific heats of fluid, air and vapor;
(4) constant heat and mass transfer coefficients and Lewis factor along the heat exchanger surface;
(5) spray water flows in a closed cycle;
(6) humidity ratio of air in equilibrium with water surface is assumed to be a linear function of the water surface temperature and the relation is given by

$$w_{\rm asw} = d + et_{\rm w}; \tag{1}$$

(7) interface temperature is assumed to be the bulk water temperature.

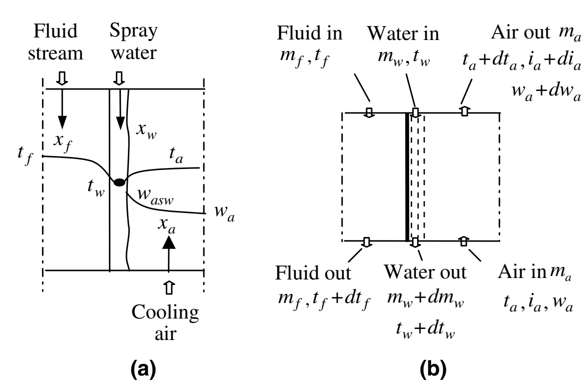


Fig. 1. Physical model of indirect evaporative cooling units: (a) schematic of the exchanger and (b) a differential element.

The errors introduced by condition (6) can be minimized by choosing appropriate values of constants d and e in Eq. (1) to give an approximate least squares fit to the true or actual saturation line over the range of water surface temperatures. The effect of condition (7) on the predicted performance will be negligible because of very large heat transfer coefficients between water film and air–water interface [5,13].

3. Differential equations

By principles of energy and mass conservation, a set of differential equations can be obtained for a differential element as shown in Fig. 1(b) as follows:

Energy balance equation for cooling air

$$m_a\,di_a = [h(t_w - t_a) + i_v h_D(w_{asw} - w_a)\sigma]A_a\,d\bar{x}_a. \tag{2}$$

Mass balance equation for cooling air and water

$$m_a\,dw_a = h_D(w_{asw} - w_a)\sigma A_a\,d\bar{x}_a \quad \text{and} \quad dm_w = -m_a\,dw_a. \tag{3}$$

Energy balance equation for fluid stream

$$m_f c_{pf}\,dt_f = U_0(t_w - t_f)A_f\,d\bar{x}_f. \tag{4}$$

Energy balance equation for the differential element

$$m_w c_{pw}\,dt_w + c_{pw}t_w\,dm_w + m_f c_{pf}\,dt_f + m_a\,di_a = 0. \tag{5}$$

For specific enthalpy of moist air, the following equation applies

$$i_a = (c_{pa} + w_a c_{pv})t_a + w_a h^0_{fg}. \tag{6}$$

In above equations, $d\bar{x}_a = dx_a/L_a$, $d\bar{x}_w = dx_w/L_w$ and $d\bar{x}_f = dx_f/L_f$ represent differential dimensionless coordinates with respect to the flow lengths. For the parallel/counterflow configurations, we have $L_a = L_w = L_f$.

After rearrangement of Eqs. (2)–(5), we can get the following set of equations:

$$dt_a = \frac{1}{B_1}(t_w - t_a)NTU\,d\bar{x}_a, \tag{7}$$

$$dw_a = (w_{asw} - w_a)\frac{\sigma}{Le_f}NTU\,d\bar{x}_a, \tag{8}$$

$$dt_f = (t_w - t_f)\frac{r}{C^*_f}NTU\,d\bar{x}_f, \tag{9}$$

$$dt_w = \left\{\frac{r}{C^*_w}(t_f - t_w) - \frac{1}{C^*_w}(t_w - t_a) - \frac{B_2\bar{h}_{fg}}{C^*_w}\frac{\sigma}{Le_f}(w_{asw} - w_a)\right\}NTU\,d\bar{x}_w \tag{10}$$

From Eq. (1), we get

$$dw_{asw} = e\,dT_w. \tag{11}$$

In above equations, $B_1 = (1 + w_a R_{cv})\Big/\left[1 + R_{cv}\frac{\sigma}{Le_f}(w_{asw} - w_a)\right]$. and $B_2 = 1 + t_w(R_{cv} - R_{cw})/\bar{h}_{fg}$. Both B_1 and B_2 are approximately equal to unity. The definition of the other grouped parameters are given as following:

$NTU = (hA_a)/(m_a c_{pa})$—represents the number of sensible heat transfer units for the cooling air side;
$r = \frac{U_0}{h}\frac{A_f}{A_a}$—represents a ratio of sensible heat transfer capacitances;
$C^*_w = \frac{m_w c_{pw}}{m_a c_{pa}}$ and $C^*_f = \frac{m_f c_{pf}}{m_a c_{pa}}$—represent water to dry air and fluid to dry air heat capacity rate ratios, respectively;
$\bar{h}_{fg} = h^0_{fg}/c_{pa}$—a normalized heat of evaporation at reference condition (0 °C);
$R_{cv} = c_{pv}/c_{pa}$ and $R_{cw} = c_{pw}/c_{pa}$—water vapor and liquid water to dry air specific heat capacity ratios, respectively;
$Le_f = h/(h_D c_{pa})$—Lewis factor for air water mixture (for simplicity and consistency, the definition of Le_f is slightly different from that in conventional practice, where $Le_{f,c}$ is defined as $h/h_D c_{pm}$. The relation between these two different definition is $Le_f = Le_{f,c}/(1 + w_a R_{cv})$).

Now, we will focus our attention on parallel/countercurrent flow configurations. Four possible flow arrangements can be imagined. These are shown in Fig. 2. Channel length for each flow stream is identical, but the dimensionless coordinates may be different in sign.

Let us define dimensionless temperature as

$$\vartheta = t/\bar{h}_{fg}. \tag{12}$$

Further, define three new grouped parameters as

$$\Delta\vartheta_{fw} = \vartheta_f - \vartheta_w, \quad \Delta\vartheta_{wa} = \vartheta_w - \vartheta_a \quad \text{and} \quad \Delta w_{wa} = w_{asw} - w_a. \tag{13}$$

Using these definitions to rearrange Eqs. (7)–(11), we can get the following set of differential equations:

$$\frac{d}{dNTU_x}\mathbf{Y} = \mathbf{AY}. \tag{14}$$

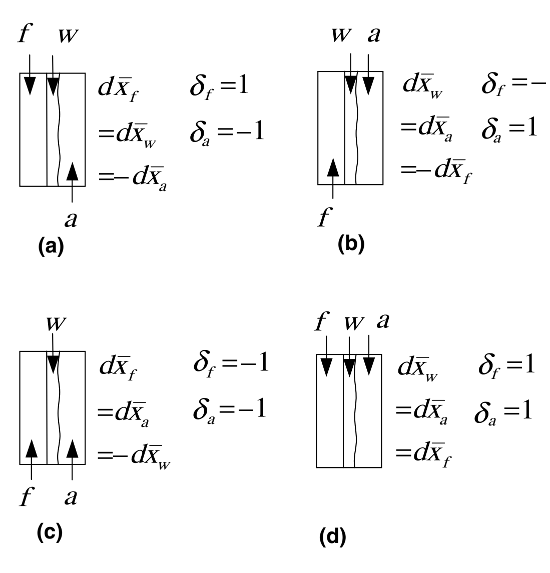


Fig. 2. Four different flow arrangements in indirect evaporative cooler with parallel/countercurrent flow configurations: a—air stream, f—primary air or process fluid, w—spray water.

In this matrix equation, $dNTU_x = NTU\,d\bar{x}_w$, the variable vector **Y** represents a set of newly defined parameters in Eq. (13)

$$\mathbf{Y} = (\Delta\vartheta_{fw}, \Delta\vartheta_{wa}, \Delta w_{wa})^T, \tag{15}$$

and **A** represents the coefficients matrix

$$\mathbf{A} = (a_{ij})_{3\times3} = \begin{pmatrix} -\left(\frac{r}{C_f^*}\delta_f + \frac{r}{C_w^*}\right) & \frac{1}{C_w^*} & \frac{B_2\sigma}{C_w^* Le_f} \\ \frac{r}{C_w^*} & -\left(\frac{1}{C_w^*} + \frac{1}{B_1}\delta_a\right) & -\frac{B_2\sigma}{C_w^* Le_f} \\ \frac{r}{C_w^*}e\bar{h}_{fg} & -\frac{1}{C_w^*}e\bar{h}_{fg} & -\frac{\sigma}{Le_f}\left(\frac{eB_2\bar{h}_{fg}}{C_w^*} + \delta_a\right) \end{pmatrix}. \tag{16}$$

In Eq. (16), $\delta_a = d\bar{x}_a/d\bar{x}_w$ and $\delta_f = d\bar{x}_f/d\bar{x}_w$, while δ_a and δ_f are flow direction indicators for cooling air and fluid stream, respectively, and will be equal to ±1, depending on the flow arrangements in the heat exchanger as shown in Fig. 2.

4. Discussion with extremity conditions

For large mass flow rate of sprayed water with $C_w^* \to \infty$, Eq. (16) will be reduced as

$$\mathbf{A} = (a_{ij})_{3\times3} = \begin{pmatrix} -\frac{r}{C_f^*}\delta_f & 0 & 0 \\ 0 & -\frac{\delta_a}{B_1} & 0 \\ 0 & 0 & -\frac{\sigma}{Le_f}\delta_a \end{pmatrix}. \tag{17}$$

That means the differential equations of $\Delta\vartheta_{fw}$, $\Delta\vartheta_{wa}$, Δw_{wa} as represented by Eqs. (14)–(16) can be integrated independently. Because of the infinitely large mass flow rate of spray water, its temperature variation along the heat exchanger surface will actually be negligible. This is equivalent to the constant spray water temperature assumption as adopted in some previous studies as reviewed in Section 1, where $\sigma/Le = 1$ was additionally assumed as unity.

For the cases with negligible water mass flow rate ($C_w^* \approx 0$) (or stationary water film) assumption, effect of enthalpy change of spray water can be neglected and Eq. (10) should be rewritten as

$$r(t_f - t_w) - (t_w - t_a) - B_2\bar{h}_{fg}\frac{\sigma}{Le_f}(w_{asw} - w_a) = 0. \tag{18}$$

Using Eq. (1) to eliminate w_{asw} in Eq. (18) and solving the resultant equation for t_w give the following equation:

$$t_w = \left[rt_f + t_a + B_2\bar{h}_{fg}\frac{\sigma}{Le_f}(w_a - d)\right] \Big/ (r + B_3), \tag{19}$$

where $B_3 = 1 + B_2e\bar{h}_{fg}\sigma/Le_f$. Using Eqs. (1) and (19) to eliminate w_{asw} and t_w in Eqs. (7)–(9) and rewriting the resultant equations in dimensionless form will give another set of differential equations. For the simplification of presentation, the same form of Eq. (14) will be utilized. However, the variable vector **Y** will be redefined as

$$\mathbf{Y} = (\vartheta_f, \vartheta_a, w_a - d)^T, \tag{20}$$

and the coefficients matrix **A** is as following

$$\mathbf{A} = (a_{ij})_{3\times3} = \frac{1}{r + B_3}\begin{pmatrix} -\frac{r}{C_f^*}B_3\delta_f & \frac{r}{C_f^*}\delta_f & B_2\frac{r}{C_f^*}\frac{\sigma}{Le_f}\delta_f \\ \frac{r}{B_1}\delta_a & \frac{1-r-B_3}{B_1}\delta_a & \frac{B_2}{B_1}\frac{\sigma}{Le_f}\delta_a \\ e\bar{h}_{fg}\frac{\sigma}{Le_f}r\delta_a & e\bar{h}_{fg}\frac{\sigma}{Le_f}\delta_a & -(r+1)\frac{\sigma}{Le_f}\delta_a \end{pmatrix}. \tag{21}$$

With further assumption of $\sigma/Le_f = 1$, Eqs. (14), (20) and (21) can be further rearranged at the approximation of $B_1 \approx 1$ and $B_2 \approx 1$ to get

$$d(\Delta\vartheta'_{fa}) = -\Delta\vartheta'_{fa}\frac{r}{r + \xi}\left(\frac{\xi}{C_f^*}\delta_f + \delta_a\right)NTU\,d\bar{x}_w, \tag{22}$$

where $\Delta\vartheta'_{fa} = \vartheta_f - \vartheta'_a$, $\vartheta'_a = (\vartheta_a + w_a - d)/\xi$ and $\xi = 1 + e\bar{h}_{fg}$. Approximately, if specific enthalpy of moist air is approximately calculated as $c_{pa}t + h^0_{fg}w_a$ and c_{pm} is approximated as c_{pa}, ϑ'_a can be considered as the dimensionless wet bulb temperature of cooling air and ξ is the ratio of the wet bulb specific heat to the dry bulb specific heat of cooling air and also the ratio of wet bulb heat transfer coefficient to the dry bulb heat transfer coefficient. Under the same approximations as stated above, the model represented by Eq. (22), Maclaine-cross and Banks' model [2] and the model used by Stabat and Marchio [11] will in effect be identical.

For general situations, the water mass flow rate should be considered as a finite value and its temperature variation cannot be neglected. Also, the grouped parameter σ/Le_f may not necessarily be equal to unity. For simplification of analysis, all the elements in the coefficients matrix can be considered as constant and be approximated with the averaged values during the whole heat and mass transfer processes. By this approximation, Eq. (14) represents a set of linear and homogeneous ordinary equations and can be solved analytically.

5. Analytical method

For Eq. (14), the characteristic equation is as following:

$$|\lambda\mathbf{E} - \mathbf{A}| = 0. \tag{23}$$

Within the practical range of the operating conditions, numerical calculation shows that the solution of this characteristic equation will give three different real roots. Thus, the analytical solution of Eq. (14) can be expressed as following:

$$\mathbf{Y} = \mathbf{K}\left(C_1e^{\lambda_1 NTU_x}, C_2e^{\lambda_2 NTU_x}, C_3e^{\lambda_3 NTU_x}\right)^T. \tag{24}$$

The elements of coefficients matrix $\mathbf{K} = (k_{ij})_{3\times3}$ can be determined by the following equation:

$$(\lambda_i\mathbf{E} - \mathbf{A})(k_{1i}, k_{2i}, k_{3i})^T = \mathbf{0}. \tag{25}$$

By satisfying Eq. (24) to the top boundary condition, i.e., for $NTU_x = 0$, $\mathbf{Y} = \mathbf{Y}_T$, we can get

$$(C_1, C_2, C_3)^T = \mathbf{K}^{-1}\mathbf{Y}_T. \tag{26}$$

Here $\mathbf{Y}_T = (\Delta\vartheta_{fw,T}, \Delta\vartheta_{wa,T}, \Delta w_{wa,T})^T$ for finite spray water mass flow rate conditions and $\mathbf{Y}_T = (\vartheta_{f,T}, \vartheta_{a,T}, w_{a,T} - d)^T$ for negligible water mass flow rate conditions. By substituting Eq. (26) into Eq. (24), we get

$$\mathbf{Y} = \mathbf{Ke}(NTU_x)\mathbf{K}^{-1}\mathbf{Y}_T, \tag{27}$$

where

$$\mathbf{e}(NTU_x) = \begin{pmatrix} e^{\lambda_1 NTU_x} & & \mathbf{0} \\ & e^{\lambda_2 NTU_x} & \\ \mathbf{0} & & e^{\lambda_3 NTU_x} \end{pmatrix}. \tag{28}$$

Setting $NTU_x = NTU$ in Eq. (27), we can get the function vector at the heat exchanger's bottom position as

$$\mathbf{Y}_B = \mathbf{B}\mathbf{Y}_T, \tag{29}$$

where the coefficients matrix $\mathbf{B}$ is defined as

$$\mathbf{B} = (b_{ij})_{3\times 3} = \mathbf{Ke}(NTU)\mathbf{K}^{-1}. \tag{30}$$

For finite mass flow rate of spray water flowing in a closed cycle, the inlet temperature will be equal to the outlet temperature. That is, $\vartheta_{w,i} = \vartheta_{w,o}$. And in consequence, $w_{asw,i} = w_{asw,o}$. With known inlet conditions for the fluid and cooling air streams, Eq. (29) correlates five unknown variables $\vartheta_{f,o}$, $\vartheta_{w,o}$, $w_{asw,o}$, $\vartheta_{a,o}$ and $w_{a,o}$ in three linear algebraic equations. In order to solve this equation to get the actual values of these variables, other confinements should be imposed. Rewriting Eq. (1) in dimensionless form can give the following equation:

$$w_{asw} = d + e\bar{h}_{fg}\vartheta_w. \tag{31}$$

We further use Eqs. (7)–(9) to rewrite Eq. (10) as following:

$$C_w^* \, dt_w = -C_f^*\delta_f \, dt_f - B_1\delta_a \, dt_a - B_2\bar{h}_{fg}\delta_a \, dw_a. \tag{32}$$

Taking all the coefficients in this equation as constants and integrating it from top to bottom or from top to any local position give the overall energy balance equations. Rewriting the resultant equations in dimensionless form gives the following equations, respectively:

For overall energy balance from top to bottom positions of the heat exchanger unit

$$C_w^*(\vartheta_{w,o} - \vartheta_{w,i}) + C_f^*(\vartheta_{f,o} - \vartheta_{f,i}) + B_1(\vartheta_{a,o} - \vartheta_{a,i}) + B_2(w_{a,o} - w_{a,i}) = 0. \tag{33}$$

For overall energy balance from top to local positions of the heat exchanger unit

$$C_w^*(\vartheta_{w,x} - \vartheta_{w,T}) + C_f^*\delta_f(\vartheta_{f,x} - \vartheta_{f,T}) + B_1\delta_a(\vartheta_{a,x} - \vartheta_{a,T}) + B_2\delta_a(w_{a,x} - w_{a,T}) = 0. \tag{34}$$

By substituting Eq. (31) for the outlet position, $w_{asw,o}$ can be eliminated from Eq. (29). Combining the resultant equations with Eq. (33), the following form of equations can be obtained after some rearrangements

$$\mathbf{F}(\vartheta_{f,o}, \vartheta_{w,o}, \vartheta_{a,o}, w_{a,o})^T = \mathbf{g}, \tag{35}$$

where the expressions for coefficients matrix $\mathbf{F}$ and vector $\mathbf{g}$ are given in Appendix A.

With negligible spray water mass flow rate condition (or with stationary water film assumption), Eq. (29) correlates three unknown variables $\vartheta_{f,o}$, $\vartheta_{a,o}$ and $w_{a,o}$ in three linear algebraic equations and can be rearranged to give the following equation:

$$\mathbf{P}(\vartheta_{f,o}, \vartheta_{a,o}, w_{a,o} - d)^T = \mathbf{q}, \tag{36}$$

where the expressions for coefficients matrix $\mathbf{P}$ and vector $\mathbf{q}$ are given in Appendix B.

6. Solution and comparison

For a given set of control parameters (for example, with known values of $t_{f,i}$, $t_{a,i}$, $w_{a,i}$, $C_{w,i}^*$, C_f^*, r, σ/Le_f and NTU), it will still be necessary to evaluate the averaged values of constants and coefficients d, e, B_1, B_2 and B_3 involved in the analytical model. These constants and coefficients will also depend on the analytical results of outlet parameters and thus 3–5 steps in iteration will be needed for the analytical solution. In evaluating constants d and e, Maclaine-cross and Banks' suggestion [2] was utilized to give an approximate least squares fit to actual saturation line over the range of water surface temperatures. That is,

$$e = (w_{asw,max} - w_{asw,min})/(t_{w,max} - t_{w,min}), \tag{37}$$

$$d = (2(w_{asw,min} + w_{asw,mean}) - w_{asw,max})/3 - et_{w,min}. \tag{38}$$

Hence, $t_{w,min}$ and $t_{w,max}$ were to be evaluated either. For indirect evaporative cooling with finite mass flow rate of recirculation water, inlet spray water temperature will be equal to the outlet spray water temperature. From practice of numerical simulation, it is easy to find that one of the two extremities $t_{w,min}$ and $t_{w,max}$ of spray water temperature occur at the inlet and outlet positions and the other extremity will occur near the inlet position. For inlet and outlet positions, spray water temperature will be given by analytical solution. For the other extremity which occurs near the inlet position, the approximate value can be obtained by solving Eq. (18) with given values of parameters t_f, t_a and w_a at top position. For indirect evaporative cooling with negligible mass flow rate of recirculation water or with stationary water film assumption, both extremities $t_{w,min}$ and $t_{w,max}$ of spray water temperature occur at the inlet and outlet positions, respectively, and their values can be obtained by solving Eq. (18) with given values of parameters t_f, t_a and w_a at the top and bottom positions, respectively. In above calculations, outlet temperatures $t_{f,o}$, $t_{a,o}$ and humidities $w_{a,o}$ can be obtained by analytical solutions. For preliminary evaluation, however, analytical results of outlet temperatures $t_{w,o}$, $t_{f,o}$, $t_{a,o}$ and humidities $w_{a,o}$ are not available and the two extremities $t_{w,min}$ and $t_{w,max}$ of spray water temperature have to be evaluated in another way. In this occasion, the two extremities can be given by letting $t_{w,min} = (t_{f,i} + t_{a,i})/2$ and $t_{w,max} = t_{w,min} + 1$. *Though this is rather arbitrarily an evaluation of the two extremities, it will not affect the final results of the iteration solution. In evaluating the averaged values of* B_1, B_2 and B_3, the arithmetic mean values of temperatures and humidities at extremity conditions were used in their expressions.

Based on the given conditions and the evaluated constants and coefficients, procedures for the analytical solution are as follows:

(1) Calculate values of the elements of coefficients matrix **A** according to Eq. (16) or Eq. (21).
(2) Calculate values of the elements of coefficients matrix **K** according to Eq. (25).
(3) Calculate values of the elements of coefficients matrix **B** according to Eq. (30).
(4) Calculate values of the elements of coefficients matrix **F** and vector **g** according to equations given in Appendix A or calculate values of the elements of coefficients matrix **P** and vector **q** according to equations given in Appendix B.
(5) Solve Eq. (35) or Eq. (36) to get the dimensionless outlet parameters and using the definition of these dimensionless parameters to get the values of the original variables.
(6) If parameter distribution needs to be calculated, Eq. (27) should only be solved explicitly using dimensionless parameters vector $\mathbf{Y}_T$ as input parameters, which can be determined from given values or analytical results of parameters at top position. This solution gives the dimensionless parameters defined in Eq. (15) or Eq. (20) at any local positions. For negligible mass flow rate of spray water, the original parameters to be solved can be directly obtained from these dimensionless parameters by simple calculations. For finite mass flow rate of spray water, this solution results should be further combined with Eq. (34) to get the dimensionless values of original variables.

Results of analytical model and numerical solutions of the one-dimensional differential equations (7)–(10) for some typical conditions were compared. In analytical model, the assumptions of linear relation of saturation humidity with water surface temperature and constant coefficients d, e, B_1, B_2 and B_3 were adopted. These are the only differences between the analytical model and the model described by the one-dimensional differential equations (7)–(10) where saturation humidity can be calculated from an accurately fitted equation of the published data [15]. In comparison, the variation range of values of the various control parameters $t_{f,i}$, $t_{a,i}$, $w_{a,i}$, $C^*_{w,i}$, C^*_f, r, σ/Le_f and NTU were selected so as to approximately cover the different conditions addressed in many literatures [2,3,5–14]. Among them, the value of the variable σ/Le_f was selected to vary from 0.5 to 1.1. To justify the exact range of the variable σ/Le_f for all application cases is rather complicated a task and perhaps deserves more detailed investigation in the future. Quite arbitrarily, Kettleborough and Hsieh [3] selected the value of the variable σ to be varied from 0 to 1 to study its influence on the heat exchanger's performance. In addition, a typical value of 0.6 was selected to study the influence of inlet air temperatures on the heat exchanger's performance and another typical value of 0.8 was select for the variable σ to investigate a real heat exchanger's performance. In practice, the value of the wettability factor will vary from case to case and be dependent on real operating conditions. On the other hand, as for the typical value of Lewis factor Le_f, the well known Lewis relation $Le_f = Le^{2/3}$ can be used, here Lewis number Le is typically considered as equal to 0.87 at

Table 1
Comparison of outlet parameters from analytical model and numerical integration for flow arrangement of Fig. 2(a)

Given conditions								Analytical results			Numerical integration		
$t_{f,i}$ (°C)	$t_{a,i}$ (°C)	$w_{a,i}$ (kg/kg(a))	$C^*_{w,i}$	$C^*_{f,i}$	r	σ/Le_f	NTU	$t_{f,o}$ (°C)	$t_{a,o}$ (°C)	$w_{a,o}$ (kg/kg(a))	$t_{f,o}$ (°C)	$t_{a,o}$ (°C)	$w_{a,o}$ (kg/kg(a))
36	30	0.01622	0	3	1	0.8	3	30.36	28.36	0.0239	30.35	28.35	0.0239
36	30	0.01622	0	3	1	0.8	1	33.46	28.27	0.0201	33.45	28.26	0.0201
36	30	0.01622	0	3	1	0.5	2	32.07	28.66	0.02166	32.07	28.65	0.02166
36	30	0.01622	0	3	1	1.1	2	31.45	27.74	0.02281	31.45	27.74	0.02281
36	30	0.01622	0	3	2.8	0.8	2	29.31	29.66	0.02466	29.3	29.66	0.02466
36	30	0.01622	0	0.75	1	0.8	2	25.87	26.77	0.02072	25.85	26.77	0.02072
36	30	0.01622	0.5	3	1	0.8	3	30.46	28.26	0.02382	30.45	28.24	0.0238
36	30	0.01622	0.5	3	8	0.8	3	26.73	30.65	0.02744	26.7	30.63	0.0274
36	30	0.01622	1	3	1	1.0	5	28.78	28.84	0.02565	28.77	28.8	0.02562
36	30	0.01622	4	3	8	1.0	2	28.55	29.7	0.02559	28.56	29.7	0.02554
50	35	0.02171	4	3	8	1.0	2	34.97	36.86	0.0397	35.02	36.87	0.03951
21	21	0.00941	4	3	8	1.0	2	18.5	19.16	0.01325	18.49	19.16	0.01325
21	21	0.00941	4	3	8	1.0	3	18.25	18.97	0.01363	18.25	18.97	0.01362
21	21	0.00941	4	3	8	1.0	1	19.06	19.75	0.01231	19.06	19.75	0.0123
36	30	0.01622	4	12	8	1.0	2	33.11	32.05	0.02972	33.11	32.04	0.02969
36	30	0.01622	4	3	8	1.1	2	28.43	29.63	0.02576	28.45	29.63	0.02571

standard atmospheric conditions [16]. On considering the above discussions, it is reasonable to select the values of the variable σ/Le_{f} to be varied from 0.5 to 1.1 in this comparison study to demonstrate the validity of the developed analytical model. It will be trivial to discuss the cases with too low values of the variable σ/Le_{f} that may seldom be encountered in engineering applications. The variation range of other parameters were selected as follows as they were usually encountered in engineering applications or for theoretical investigation purposes [2,3,5–14]: $t_{\mathrm{f,i}} = 21$–$50\ ^{\circ}\mathrm{C}$, $t_{\mathrm{a,i}} = 21$–$35\ ^{\circ}\mathrm{C}$, $w_{\mathrm{a,i}} = 0.00941$–$0.02171$ kg/kg(a), $C^*_{\mathrm{w,i}} = 0$–4, $C^*_{\mathrm{f,i}} = 0.75$–$12$, $r = 1$–8, $NTU = 1$–5. The comparison showed good agreement between the analytical results and results by numerical integration for all the cases shown in Fig. 2. For simplification, however, only typical results for case (a) in Fig. 2 were presented in Table 1. From Table 1, relative errors, which are defined as $|(v_{\mathrm{o}}^{\mathrm{nu}} - v_{\mathrm{o}}^{\mathrm{an}})/\ (v_{\mathrm{o}}^{\mathrm{nu}} - v_{\mathrm{i}})|$—the ratios of the absolute errors in calculated values of a variable to the absolute changes of the values of the corresponding variable, can be calculated. It was found that the maximum errors were 3.5% for $t_{\mathrm{a,o}}$, 0.4% for $t_{\mathrm{f,o}}$ and 1.06% for $w_{\mathrm{a,o}}$ while the averaged errors were only 0.64% for $t_{\mathrm{a,o}}$, 0.17% for $t_{\mathrm{f,o}}$ and 0.24% for $w_{\mathrm{a,o}}$. These errors are very small for an analytical solution and the validity of the assumptions adopted in the analytical model were thus demonstrated. Though no comparison were made to be shown in Table 1 for the cases with much lower values of σ/Le_{f}, it can still be reasonably expected by logic that the model will also apply for such cases because no assumptions were made in the above analytical model that will preclude such applications.

7. Application analysis

For discussion, the performance of indirect evaporative cooler can be evaluated by the cooling effectiveness defined as follows:

$$\varepsilon = (t_{\mathrm{f,i}} - t_{\mathrm{f,o}})/(t_{\mathrm{f,i}} - t_{\mathrm{wb,i}}). \tag{39}$$

As application examples, parameter distributions were calculated using the analytical model for the four different flow arrangements as shown in Fig. 2 under some typical conditions as $t_{\mathrm{f,i}} = 36\ ^{\circ}\mathrm{C}$, $t_{\mathrm{a,i}} = 30\ ^{\circ}\mathrm{C}$, $w_{\mathrm{a,i}} = 0.01622$ kg/kg(a), $C^*_{\mathrm{w,i}} = 0$, 0.7 or 5, $C^*_{\mathrm{f}} = 3$, $r = 8$, $\sigma/Le_{\mathrm{f}} = 1$, 0.7 or 1 and $NTU = 3$. Results were presented in Figs. 3 and 4. Also presented on these figures were the results of numerical simulations based on the one-dimensional differential equations (7)–(10). It was found that the analytical and numerical results were in very good agreement. Thus, the analytical model was further demonstrated to be valid in determining parameter distributions.

Fig. 3 shows the parameter distributions for the flow arrangement of Fig. 2(a) with different values of C^*_{w} and σ/Le_{f}. Due to counter flow arrangements, the fluid stream is cooled and its temperature drops continuously while the wet bulb temperature of air is heated up during the heat and mass transfer processes. The alternation of sensible heat transfer direction between spray water film and secondary air causes the dry bulb temperature of air to drop at first and then increases again. For finite spray water mass flow rate conditions as shown in Fig. 3(b) and (c), the inlet effect of spray water will also affect the performances of the heat exchanger. For recirculation, inlet spray water temperature will be equal to its outlet value and thus will be heated up quickly in the entrance region. This inlet effect will accelerate the cooling of the fluid stream in the entrance region and may cool down secondary air dry bulb and wet bulb temperature by the exit position. After that, the spray water temperature will again be cooled down to its outlet value as in the cases with negligible spray water mass flow rate conditions as shown in Fig. 3(a). Through

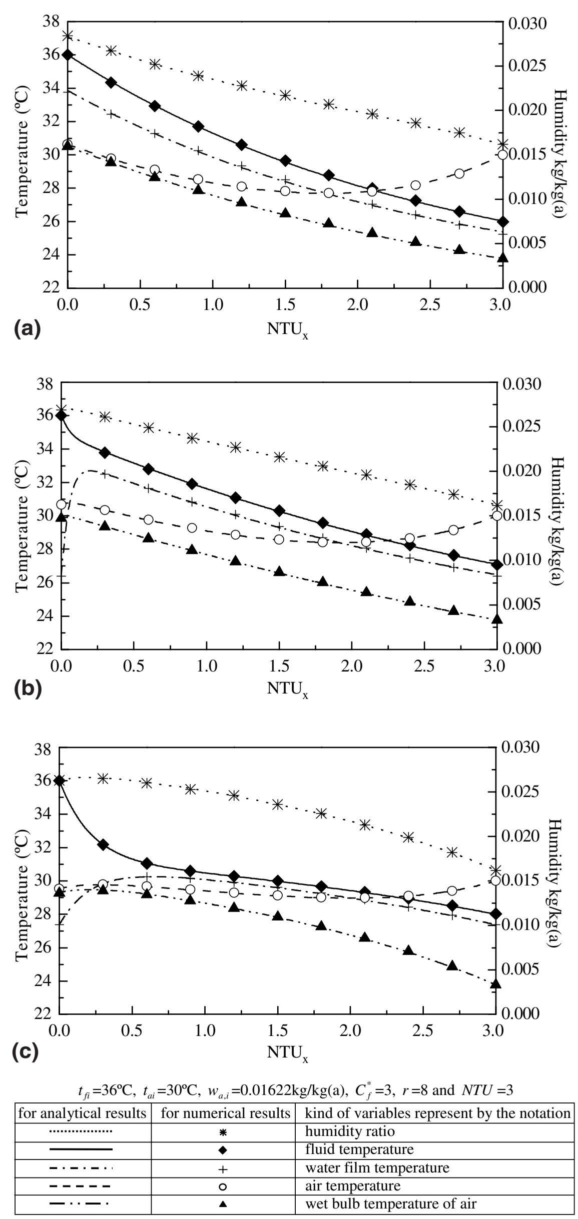
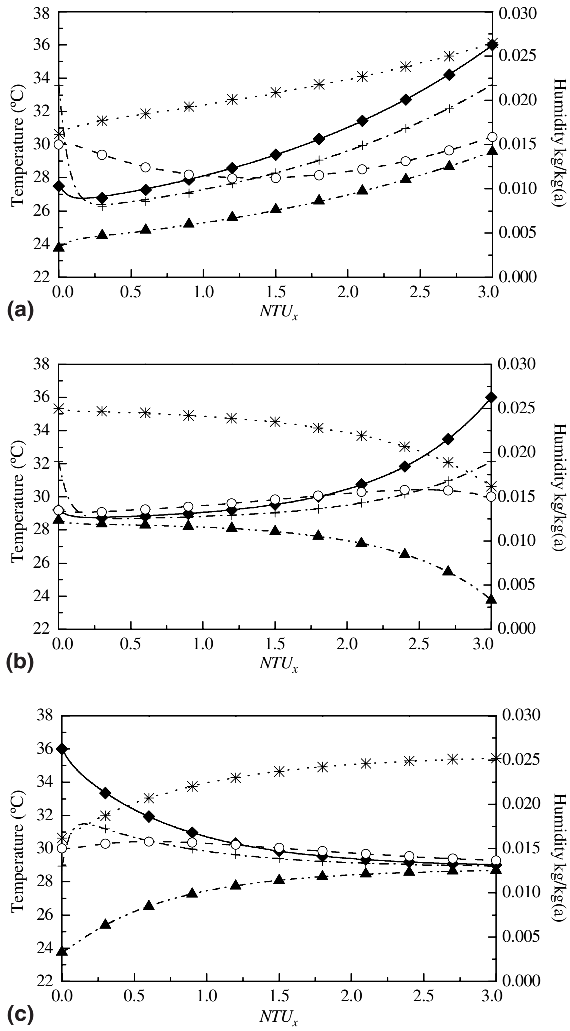


Fig. 3. Parameter distribution for flow arrangement of Fig. 2(a) under some typical conditions. For $C^*_{\mathrm{w,i}} = 0$, $\sigma/Le_{\mathrm{f}} = 1$ (a), $C^*_{\mathrm{w,i}} = 0.7$, $\sigma/Le_{\mathrm{f}} = 0.7$ (b), and $C^*_{\mathrm{w,i}} = 5$, $\sigma/Le_{\mathrm{f}} = 1$ (c).



t_{fi}=36ºC, t_{ai}=30ºC, $w_{a,i}$=0.01622kg/kg(a), $C^*_{w,i}$=0.7, C^*_f=3, r=8, σ / Le_f=0.7, NTU=3

for analytical results	for numerical results	kind of variables represent by the notation
················	✱	humidity ratio
————	◆	fluid temperature
— · — · — ·	+	water film temperature
– – – – –	○	air temperature
—· · —· · —	▲	wet bulb temperature of air

Fig. 4. Parameter distribution for different flow arrangements under a typical condition. (a) Case (b) in Fig. 2, (b) case (c) in Fig. 2, and (c) case (d) in Fig. 2.

comparison, it was found that the greatest fluid temperature reduction occurred under the idealized conditions with negligible spray water mass flow rate ($C^*_{w,i} = 0$) and almost complete wetting of sprayed surface ($\sigma/Le_f = 1$) (Fig. 3(a)). This gave the highest performance for the indirect evaporative cooler. For other conditions with increased spray water mass flow rates ($C^*_{w,i} = 0.7$ or 5) or decreased surface wettability ($\sigma/Le_f = 0.7$) (Fig. 3(b) and (c)), smaller fluid temperature reductions and thus lower performances were obtained. This phenomenon could be attributed to the effect of increased heat capacity of spray water or decreased mass transfer surface area for evaporation. With increased heat capacity of spray water, smaller mean temperature difference between fluid and water film results. With decreased mass transfer surface area, effect of evaporative cooling is also decreased. Both of these affecting factors leads to the decreased heat transfer performances.

Fig. 4 shows the parameter distributions for the flow arrangements of other three cases (b)–(d) in Fig. 2 with the condition of $t_{f,i} = 36$ °C, $t_{a,i} = 30$ °C, $w_{a,i} = 0.01622$ kg/kg(a), $C^*_{w,i} = 0.7$, $C^*_f = 3$, $r = 8$, $\sigma/Le_f = 0.7$ and $NTU = 3$. Generally, the wet bulb temperature of air increases and fluid temperature is cooled down during the whole heat and mass transfer processes because of the total heat transfer taking place between the air and the fluid stream. In Fig. 4(a) and (b), however, spray water flows counter currently to the fluid stream and its temperature is heated up to a maximum at its outlet positions and thus the inlet spray water temperature will also be high. Under such conditions, the inlet effect of spray water causes the fluid temperature to be heated up suddenly just by the exit. Through comparison of parameter distributions shown in Figs. 3(b) and 4, it was found that the flow arrangement of Fig. 2(a) gave better performance (higher effectiveness value) than the other three cases (Fig. 2(b)–(d)) under the same conditions. These could also be seen from performance data as shown in Table 2 that gave out the fluid outlet temperatures and effectivenesses at different conditions for the four different cases of Fig. 2. Only for negligible spray water mass flow rate conditions, case (b) gave the same performances as case (a).

8. Conclusion

The characteristic of indirect evaporative cooling under real operating conditions was discussed. Literatures were reviewed. Traditionally, models for indirect evaporative cooler with high accuracy were solved numerically and

Table 2
Performance data for different flow arrangements under some typical conditions[a]

Flow arrangements	Outlet fluid temperatures and effectivenesses					
	For $C^*_{w,i} = 0$, $\sigma/Le_f = 1$		For $C^*_{w,i} = 0.7$, $\sigma/Le_f = 0.7$		For $C^*_{w,i} = 5$, $\sigma/Le_f = 1$	
	$t_{f,o}$	ε	$t_{f,o}$	ε	$t_{f,o}$	ε
Fig. 2(a)	26.02	0.816	27.10	0.728	28.01	0.654
Fig. 2(b)	26.02	0.817	27.56	0.691	29.14	0.561
Fig. 2(c)	28.88	0.582	29.21	0.555	29.20	0.556
Fig. 2(d)	28.88	0.582	29.02	0.571	28.91	0.580

[a] $t_{f,i} = 36$ °C, $t_{a,i} = 30$ °C, $w_{a,i} = 0.01622$ kg/kg(a), $C^*_f = 3$, $r = 8$ and $NTU = 3$.

simplified methods will sacrifice accuracy for simplicity of solution. In this paper, an analytical approach was developed that combines simplicity of solution and accuracy of detailed models where nonunity values of Lewis factor, incomplete surface wetting condition, and effects of spray water evaporation, spray water temperature variation and spray water enthalpy change were taken into consideration. Through comparison, results of analytical solutions were found to be in good agreement with those of numerical integrations.

Performances with four different flow configurations were briefly discussed using the analytical model. Parameter distributions were calculated. Analytical results were still in very good agreement with the numerical results. Through comparison, it was found that for the flow arrangement of Fig. 2(a) where cooling air flows in countercurrent to the water film and fluid streams, the indirect evaporative cooler gave better performance than the other three flow arrangements shown in Fig. 2(b)–(d) where cooling air flows parallel to the fluid stream and/or water film. With negligible spray water flow rate and complete surface wetting conditions, however, both flow arrangements of Fig. 2(a) and (b) gave the same best performances where cooling air always flows countercurrent to the fluid stream. Thus, decreasing spray water mass flow rate and improving surface wettability leads to the improve performances of indirect evaporative cooler with countercurrent flow configurations for cooling air and fluid flows.

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Appendix A. Coefficients matrix F and vector g for Eq. (35)

Coefficients matrix **F** and vector **g** for Eq. (35) for the four cases as shown in Fig. 2 are given as follows:

For case (a) in Fig. 2,

$$\mathbf{F} = \begin{pmatrix} 1 & -1-D_1 & b_{12} & b_{13} \\ 0 & 1-D_2 & b_{22} & b_{23} \\ 0 & e\bar{h}_{\mathrm{fg}}-D_3 & b_{32} & b_{33} \\ C_{\mathrm{f}}^* & 0 & B_1 & B_2 \end{pmatrix} \quad \text{and} \quad \mathbf{g} = \begin{pmatrix} b_{11}\vartheta_{\mathrm{f,i}} + b_{13}d \\ \vartheta_{\mathrm{a,i}} + b_{21}\vartheta_{\mathrm{f,i}} + b_{23}d \\ -d + w_{\mathrm{a,i}} + b_{31}\vartheta_{\mathrm{f,i}} + b_{33}d \\ C_{\mathrm{f}}^*\vartheta_{\mathrm{f,i}} + B_1\vartheta_{\mathrm{a,i}} + B_2 w_{\mathrm{a,i}} \end{pmatrix}. \tag{A.1}$$

For case (b) in Fig. 2,

$$\mathbf{F} = \begin{pmatrix} -b_{11} & -1-D_1 & 0 & 0 \\ -b_{21} & 1-D_2 & -1 & 0 \\ -b_{31} & e\bar{h}_{\mathrm{fg}}-D_3 & 0 & -1 \\ C_{\mathrm{f}}^* & 0 & B_1 & B_2 \end{pmatrix} \quad \text{and} \quad \mathbf{g} = \begin{pmatrix} -\vartheta_{\mathrm{f,i}} - b_{12}\vartheta_{\mathrm{a,i}} + b_{13}(d - w_{\mathrm{a,i}}) \\ -b_{22}\vartheta_{\mathrm{a,i}} + b_{23}(d - w_{\mathrm{a,i}}) \\ -d - b_{32}\vartheta_{\mathrm{a,i}} + b_{33}(d - w_{\mathrm{a,i}}) \\ C_{\mathrm{f}}^*\vartheta_{\mathrm{f,i}} + B_1\vartheta_{\mathrm{a,i}} + B_2 w_{\mathrm{a,i}} \end{pmatrix}. \tag{A.2}$$

For case (c) in Fig. 2,

$$\mathbf{F} = \begin{pmatrix} -b_{11} & -1-D_1 & b_{12} & b_{13} \\ -b_{21} & 1-D_2 & b_{22} & b_{23} \\ -b_{31} & e\bar{h}_{\mathrm{fg}}-D_3 & b_{32} & b_{33} \\ C_{\mathrm{f}}^* & 0 & B_1 & B_2 \end{pmatrix} \quad \text{and} \quad \mathbf{g} = \begin{pmatrix} -\vartheta_{\mathrm{f,i}} + b_{13}d \\ \vartheta_{\mathrm{a,i}} + b_{23}d \\ -d + w_{\mathrm{a,i}} + b_{33}d \\ C_{\mathrm{f}}^*\vartheta_{\mathrm{f,i}} + B_1\vartheta_{\mathrm{a,i}} + B_2 w_{\mathrm{a,i}} \end{pmatrix}. \tag{A.3}$$

For case (d) in Fig. 2,

$$\mathbf{F} = \begin{pmatrix} 1 & -1-D_1 & 0 & 0 \\ 0 & 1-D_2 & -1 & 0 \\ 0 & e\bar{h}_{\mathrm{fg}}-D_3 & 0 & -1 \\ C_{\mathrm{f}}^* & 0 & B_1 & B_2 \end{pmatrix} \quad \text{and} \quad \mathbf{g} = \begin{pmatrix} b_{11}\vartheta_{\mathrm{f,i}} - b_{12}\vartheta_{\mathrm{a,i}} + b_{13}(d - w_{\mathrm{a,i}}) \\ b_{21}\vartheta_{\mathrm{f,i}} - b_{22}\vartheta_{\mathrm{a,i}} + b_{23}(d - w_{\mathrm{a,i}}) \\ -d + b_{31}\vartheta_{\mathrm{f,i}} - b_{32}\vartheta_{\mathrm{a,i}} + b_{33}(d - w_{\mathrm{a,i}}) \\ C_{\mathrm{f}}^*\vartheta_{\mathrm{f,i}} + B_1\vartheta_{\mathrm{a,i}} + B_2 w_{\mathrm{a,i}} \end{pmatrix}. \tag{A.4}$$

In Eqs. (A.1)–(A.4), $D_1 = -b_{11} + b_{12} + b_{13}e\bar{h}_{\mathrm{fg}}$, $D_2 = -b_{21} + b_{22} + b_{23}e\bar{h}_{\mathrm{fg}}$, $D_3 = -b_{31} + b_{32} + b_{33}e\bar{h}_{\mathrm{fg}}$.

Appendix B. Coefficients matrix P and vector q for Eq. (36)

Coefficients matrix **P** and vector **q** for Eq. (36) for the four cases as shown in Fig. 2 are given as follows:

For case (a) in Fig. 2,

$$\mathbf{P} = \begin{pmatrix} 1 & -b_{12} & -b_{13} \\ 0 & -b_{22} & -b_{23} \\ 0 & -b_{32} & -b_{33} \end{pmatrix} \quad \text{and} \quad \mathbf{q} = \begin{pmatrix} b_{11}\vartheta_{\mathrm{f,i}} \\ b_{21}\vartheta_{\mathrm{f,i}} - \vartheta_{\mathrm{a,i}} \\ b_{31}\vartheta_{\mathrm{f,i}} - (w_{\mathrm{a,i}} - d) \end{pmatrix}. \tag{B.1}$$

For case (b) in Fig. 2,

$$\mathbf{P} = \begin{pmatrix} -b_{11} & 0 & 0 \\ -b_{21} & 1 & 0 \\ -b_{31} & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{q} = \begin{pmatrix} -\vartheta_{\mathrm{f,i}} + b_{12}\vartheta_{\mathrm{a,i}} + b_{13}(w_{\mathrm{a,i}} - d) \\ b_{22}\vartheta_{\mathrm{a,i}} + b_{23}(w_{\mathrm{a,i}} - d) \\ b_{32}\vartheta_{\mathrm{a,i}} + b_{33}(w_{\mathrm{a,i}} - d) \end{pmatrix}. \tag{B.2}$$

For case (c) in Fig. 2,

$$\mathbf{P} = \mathbf{B} \quad \text{and} \quad \mathbf{q} = (\vartheta_{\mathrm{f,i}}, \vartheta_{\mathrm{a,i}}, w_{\mathrm{a,i}} - d)^{\mathrm{T}}. \tag{B.3}$$

For case (d) in Fig. 2,

$$\mathbf{P} = \mathbf{E} \quad \text{and} \quad \mathbf{q} = \mathbf{B}(\vartheta_{\mathrm{f,i}}, \vartheta_{\mathrm{a,i}}, w_{\mathrm{a,i}} - d)^{\mathrm{T}}. \tag{B.4}$$

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